Riddle 2!

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Before, actually getting into the original problem, let's think about an subset of the original issue. Imagine, there is "a" bus, with infinite number of new arrivals. How will the accommodate the new arrivals? We can't use the hint, that I had given, $n + \infty$, really! Something does not look right. So, we need to think of a way, that will divide the infinite number of rooms into two infinite groups, ie., odd and even rooms. So we will move the existing guests, to room $2n$. So all the existing guests, will move to all even numbered rooms, which will empty out all the odd numbered rooms, so we can accommodate the infinite number of newly arrived guests in the infinitely many odd numbered rooms, so room n is going to move to $2n$.

Now, I think you guys would have gotten a pretty good idea about the original question. We will now discuss that, By Euclid's Lemma, we know that there are infinitely many primes. So, we will use the prime numbers in the problem. So to accommodate, an infinite number of buses with an infinite number of people in each bus, we need to move everybody in the rooms, to the 1st prime no. to the power of their room no. So, mathematically, we can define the function $N_R : \mathbb{Z} \to \mathbb{Z}$, with $r \mapsto 2^r$, this is nothing fancy, N_R denotes the function "new rooms" and r denotes room no, heeee! So,

$$1 \mapsto 2^1 = 2$$
$$\vdots$$
$$5 \mapsto 2^5 = 32$$
$$\vdots$$
$$8 \mapsto 2^8 = 256$$
$$\vdots$$

So, now we took care of the current guests that were staying at the hotel by moving them into new rooms. What about the infinitely many people waiting in those infinitely many buses? Before, we get crazy with infinity, we will denote, the people waiting in those buses for a piece of mind. Let (m, n) denote the bus no., m and seat no., n for the infinitely many people in those buses. We will now move the passengers to the rooms, using the following function, $R_B : \mathbb{Z} \to \mathbb{Z}$, with $(m, n) \mapsto p_{m+1}^n$, where p_{m+1}, denotes the $(m+1)^{\text{th}}$ prime. So,

$$(1, 1) \mapsto 3^1 = 3$$
$$\vdots$$
$$(1, 7) \mapsto 3^7 = 2187$$
$$\vdots$$
$$(5, 4) \mapsto 13^4 = 28561$$
$$\vdots$$

So, this is the solution.

It is not so hard to prove it mathematically rigorous.

For fun, you can try that if there are infinitely many ships, with infinitely many buses, with infinitely many people. How will you accommodate all those passengers?