Proof of Fermat's Last Theorem in One page! What...?

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Abstract

Obviously! One could have noticed that we are not going to prove the actual Fermat's Last Theorem, rather we will prove a slight variation of the original statement (just by adding a extra condition). This condition changes the difficulty of the problem by day and night.

Statement: Let $x, y, z, n \in \mathbb{N}$ and $n \geq z$, then $x^n + y^n = z^n$ has no non-trivial integer solutions.

Proof. For the sake of contradiction, suppose $x^n + y^n = z^n$ has a non-trivial integer solution. Without the loss of generality, assume $x < y$. So, we have

$$x^n = z^n - y^n = (z - y)(z^{n-1} + yz^{n-2} + \cdots + y^{n-1})$$

Since, we have $x < z$ and $y < z$, by substituting y we have

$$\begin{aligned}(z - y)(z^{n-1} + yz^{n-2} + \cdots + y^{n-1}) &> (z - y)(y^{n-1} + yy^{n-2} + \cdots + y^{n-1}) \\ &> (z - y)ny^{n-1} \\ &> ny^{n-1}\end{aligned}$$

Also, since $n \geq z$, we have $x^n > ny^{n-1} > zy^{n-1} > zx^{n-1} > x^n$. Hence, we get a contradiction that $x^n > x^n$, which is not possible. Hence proved. □